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# Physics in extra dimensions: lecture #4

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*Lecture 1:* Field theory in compact dimensions.

Gauge bosons in the bulk and their collider signatures.

*Lecture 2:* One universal extra dimension.

**D**iscrete symmetries and cascade decays at colliders.

*Lecture 3:* Two universal extra dimensions.

*Lecture 4:* **P**articles in a warped extra dimension.

**S**trongly-coupled physics at the TeV scale

## A warped extra dimension

*L. Randall, R. Sundrum, hep-ph/9905221*

4D flat spacetime of coordinates  $x^\mu$  and one dimension of coordinate  $z$ .

Line element:

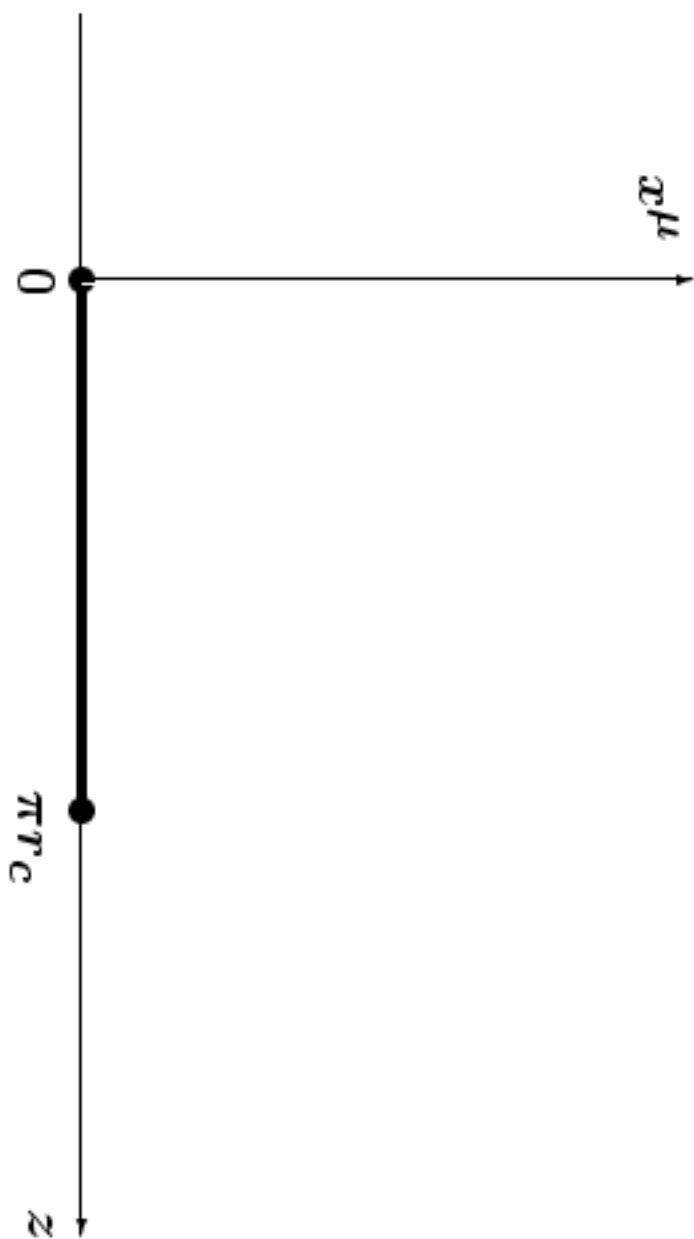
$$ds^2 = e^{-2kz} \eta_{\mu\nu} dx^\mu dx^\nu - dz^2, \quad \eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Anti-de-Sitter space along the 5th dimension.

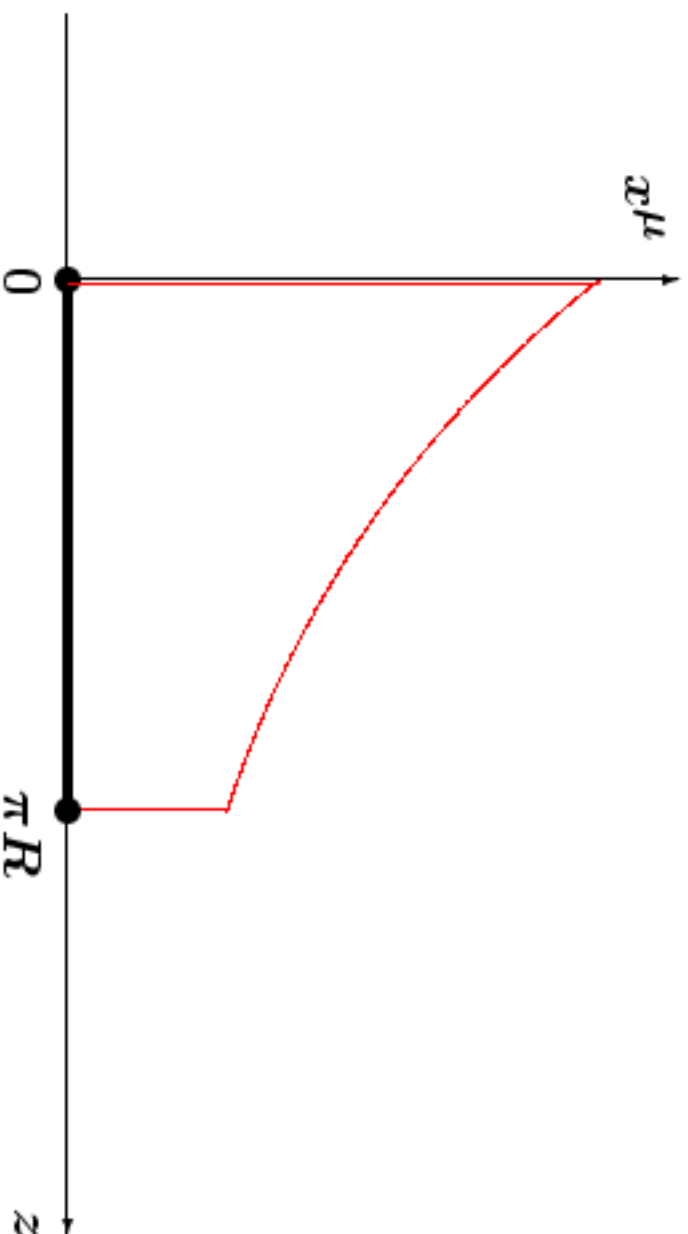
$k$  is the AdS curvature (has dimensions of mass).

*The unit of length depends on the position along  $z$ !*

**A slice of Anti-de-Sitter space: space exists only for  $0 < z < \pi r_c$ .**

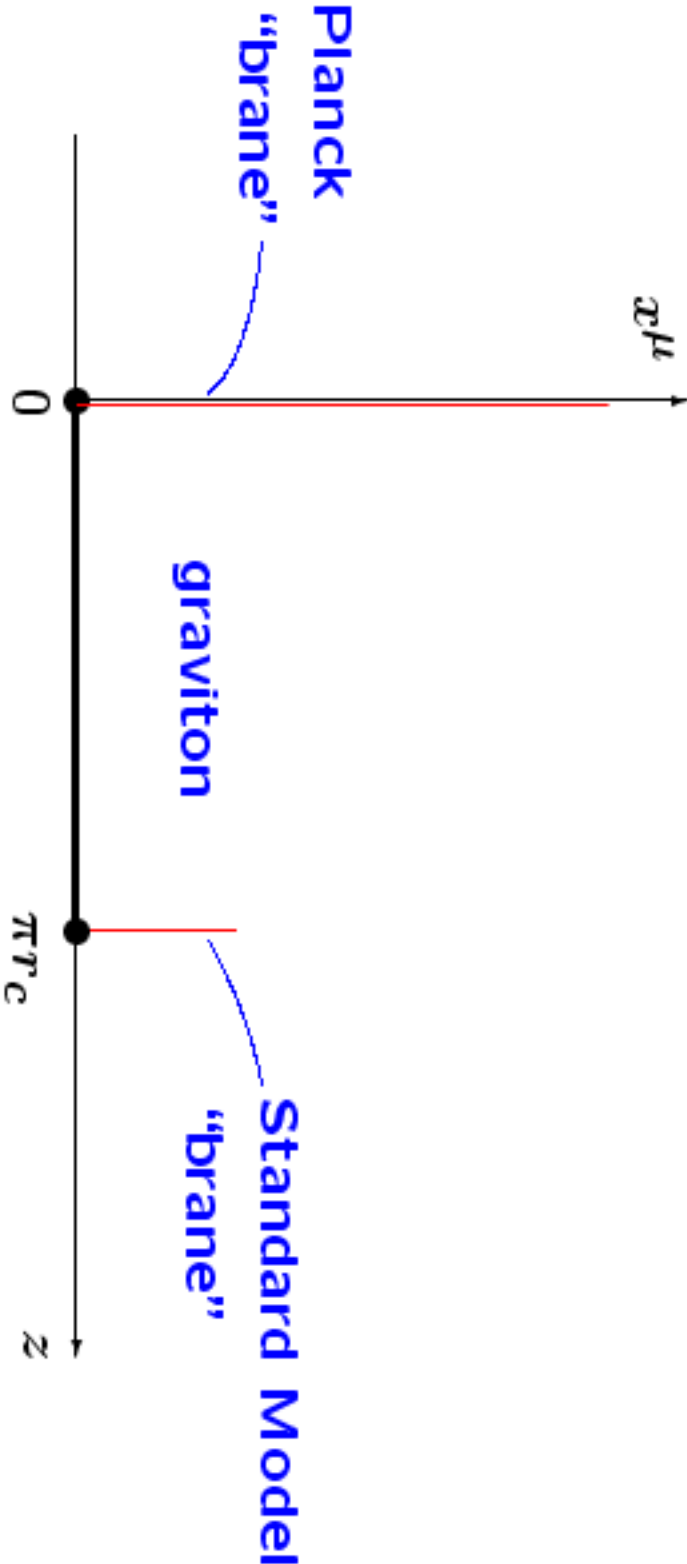


**A slice of Anti-de-Sitter space: space exists only for  $0 < z < \pi R$ .**



**Boundary conditions at  $z = 0$  and  $z = \pi R$  must be specified for each field propagating in the bulk.**

**RS1 model:**



**Scales:**  $kr_c \approx 10$  ,  $k \lesssim M_{\text{Planck}} \approx 10^{19} \text{ GeV}$

**Fluctuations of the metric:**

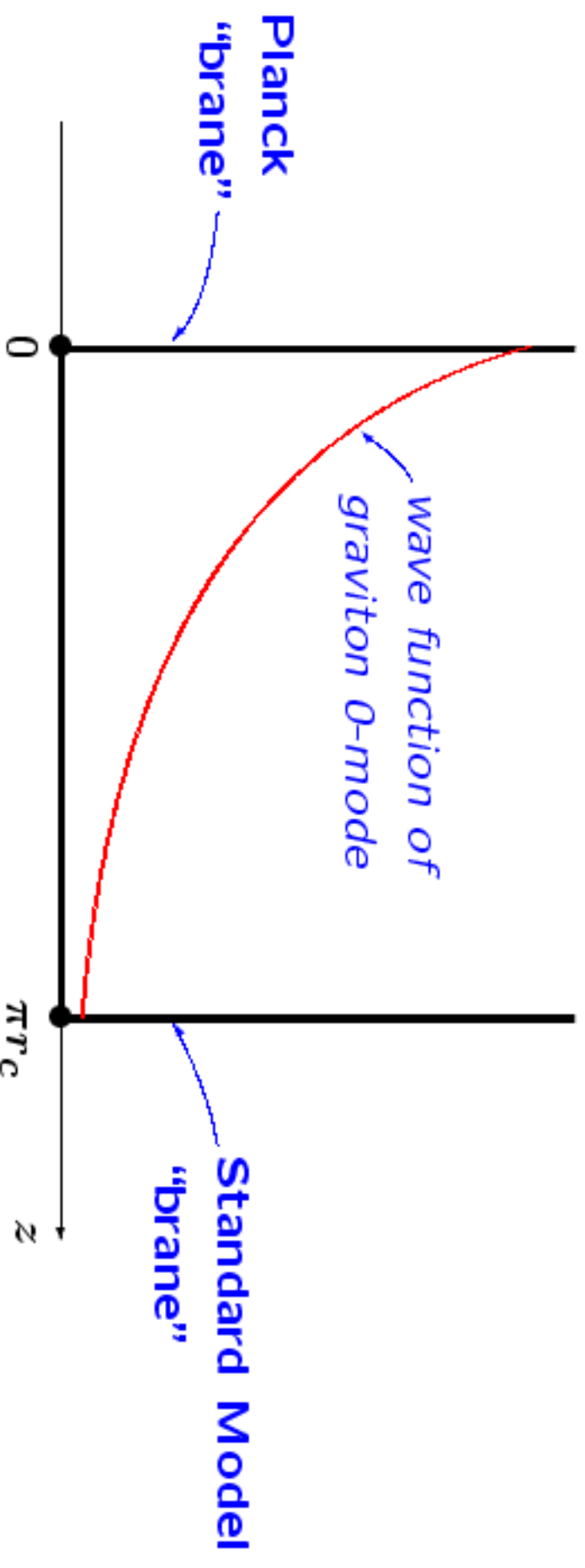
$$ds^2 = e^{-2kz} \left[ \eta_{\mu\nu} + h_{\mu\nu}(x) \right] dx^\mu dx^\nu - dz^2$$

$h_{\mu\nu}$  is the graviton zero-mode, responsible for the long-range gravitational interactions.

*Strength of gravitational force:*  $G_N \sim 1/M_{\text{Planck}}^2$

**Fundamental scale on the Standard Model brane:**

$$M_{\text{Planck}} e^{-\pi k r_c} \sim O(1) \text{ TeV}$$



Interaction of the graviton 0-mode (the massless 4D spin-2 field) with Standard Model particles is suppressed by its exponentially small wave function at the SM brane.

*Hierarchy between the Planck and weak scale is explained!*

## Comparison between various solutions to the hierarchy problem:

### 1. Technicolor

*Exponential hierarchy between  $M_{\text{Planck}}$  and the scale where the technicolor gauge interaction becomes strong.*

**Problem: fit to the electroweak data? (some solutions exist)**

### 2. Dynamically-broken supersymmetry

*Susy breaking scale is exponentially suppressed compared to  $M_{\text{Planck}}$  due to gauge dynamics.*

**Problem:  $\mu$  term (the Higgsino mass) is supersymmetric.**

**Why  $\mu \sim v$ ? (some solutions exist)**

### 3. RS1

*$1/M_{\text{Planck}}$  is exponentially suppressed compared to  $1/v$ .*

Energy

$10^{16}$  TeV

$\sim 1$  TeV

quantum gravity

**Technicolor gauge coupling:**  $g_{TC} \sim O(1)$

*logarithmic running of  $g_{TC}$   
(increases at lower scales,  
just as in QCD)*

$g_{TC} \sim O(4\pi) \Rightarrow$  **Technifermions condense**  
 $\Rightarrow$  **electroweak symmetry is broken**

**Energy**

$10^{16}$  TeV

$\sim 10 - 10^8$  TeV

$10 - 10^7$  TeV ?

$\sim 1$  TeV

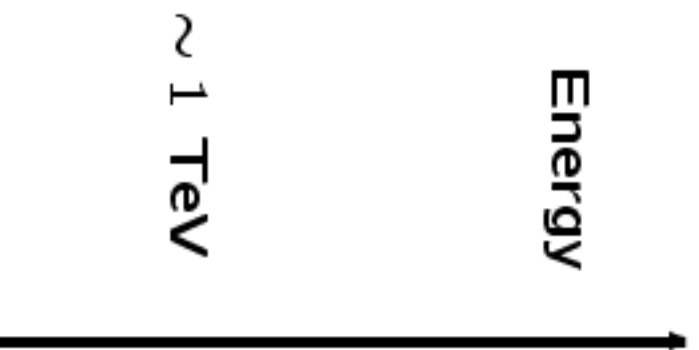
**quantum gravity**

**Dynamical susy  
breaking sector**

**Messenger sector**

**MSSM**

## Scales of RS1 model:



**Graviton KK modes are strongly coupled**

**Standard Model**

## Gauge fields in a warped extra dimension

Kaluza-Klein decomposition for the gauge fields

$$A_\mu(x^\nu, z) = \frac{1}{\sqrt{2\pi r_c}} \left[ A_\mu^{(0)}(x^\nu) + \sum_{j \geq 1} A_\mu^{(j)}(x^\nu) f_j(z) \right]$$

0-mode has a flat profile (unlike the graviton).

KK functions:

$$f_j(z) = \frac{e^{kz}}{N_j} \left[ J_1 \left( \frac{m_j}{k} e^{kz} \right) - \frac{J_0(m_j/k)}{Y_0(m_j/k)} Y_1 \left( \frac{m_j}{k} e^{kz} \right) \right]$$

$J, Y$  are Bessel functions;  $N_j$  is a normalization constant

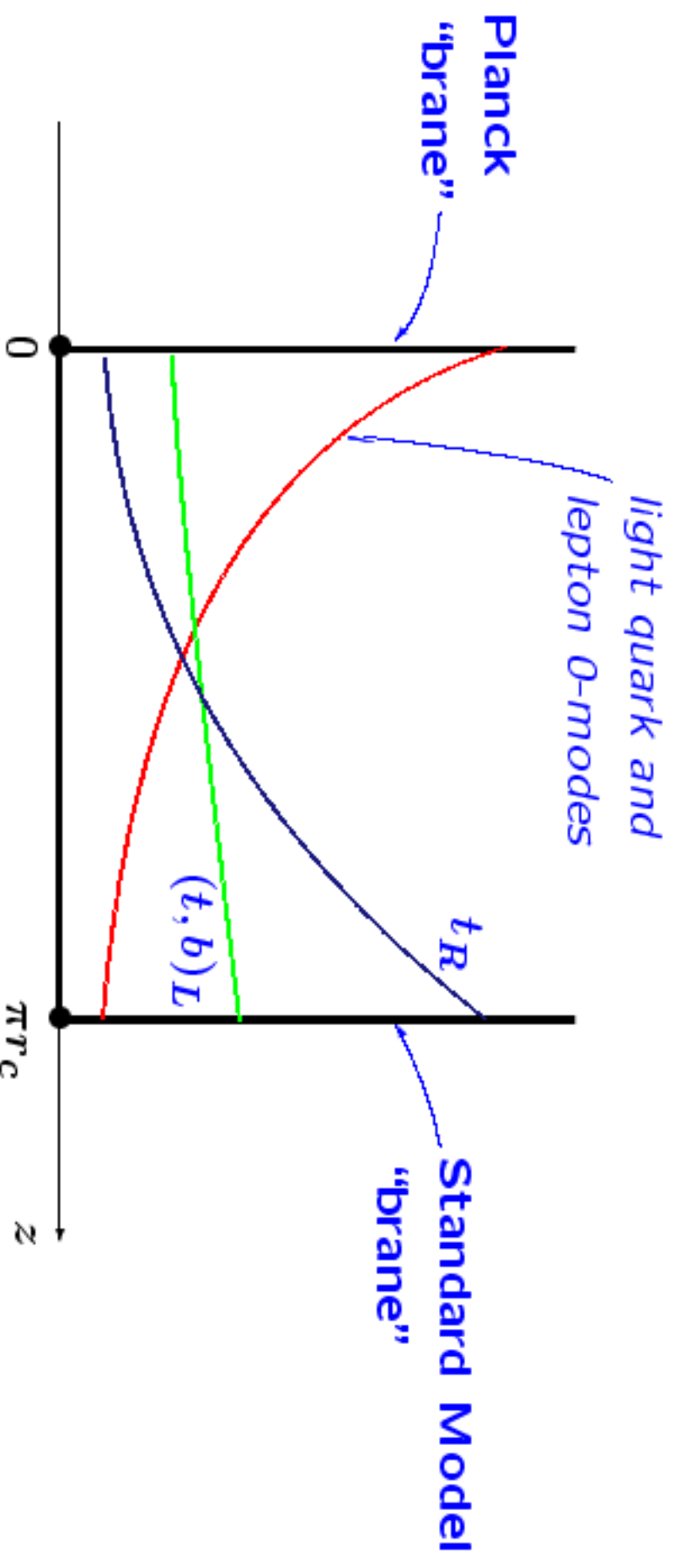
$m_j$  is the mass of the  $j$ th KK mode of spin-1:

$$m_1 = 2.5\bar{k}, \quad m_2 = 5.6\bar{k}, \quad m_3 = 8.7\bar{k}, \quad \dots$$

$$\bar{k} \equiv k e^{-kr_c\pi} \sim O(1) \text{ TeV}$$

Dirac equation in a warped dimension  $\Rightarrow$  fermion zero-modes have a non-trivial profile, peaked on one of the branes.

For each standard model fermion there is a 5D mass parameter which controls the bulk profile.



## Standard Model in a warped extra dimension:

*unlike UED, there is no KK parity because of the warping.*

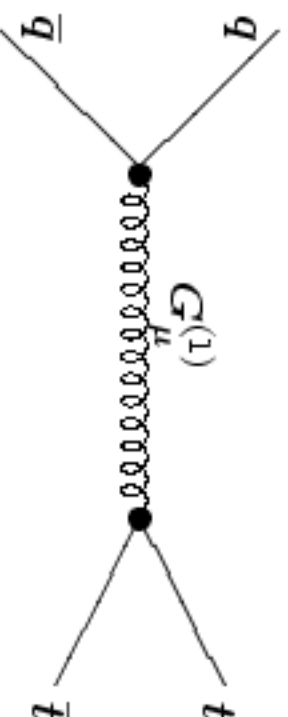
**Severe constraints on the KK masses from electroweak fits:  $m_1 > O(20)$  TeV**

*Csaki, J. Erlich and J. Terning, hep-ph/0203034; G. Burdman, hep-ph/0205329*

**Limits are lowered to  $m_1 > O(3)$  TeV if the gauge group in the bulk is  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$**

*K. Agashe, et al hep-ph/0308036*

**Typical signature at the LHC:**



**The fermion KK modes may have masses below 1 TeV**

**(Carena, Ponton, Santiago, Wagner, hep-ph/0701055)**

*Conjecture:* **SM in a warped extra dimension is dual to a 4D quasi-conformal strongly-coupled gauge theory**  
(a deformation of the AdS/CFT conjecture)

**Similar to “walking technicolor” ?!**

**Fields in the 5D picture localized close to the SM brane are composite fields in the conformal theory:**

- **Higgs doublet and  $t_R$  are composite fields**
- **$(t, b)_L$  is partially composite (an admixture of a composite field and a fundamental field).**

AdS/CFT interpretation of a warped extra dimension is yet another connection to 4D physics.

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**Recall from lecture #1:**

*5D theory = 4D theory with some heavy particles*

$SU(3)_c$  in extra dimensions  $\rightarrow$  SM gluon + heavy gluons

**4D theory describing the first  $N$  KK modes of the gluon:**

$SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_{N+1} \rightarrow SU(3)_c$  gauge group,

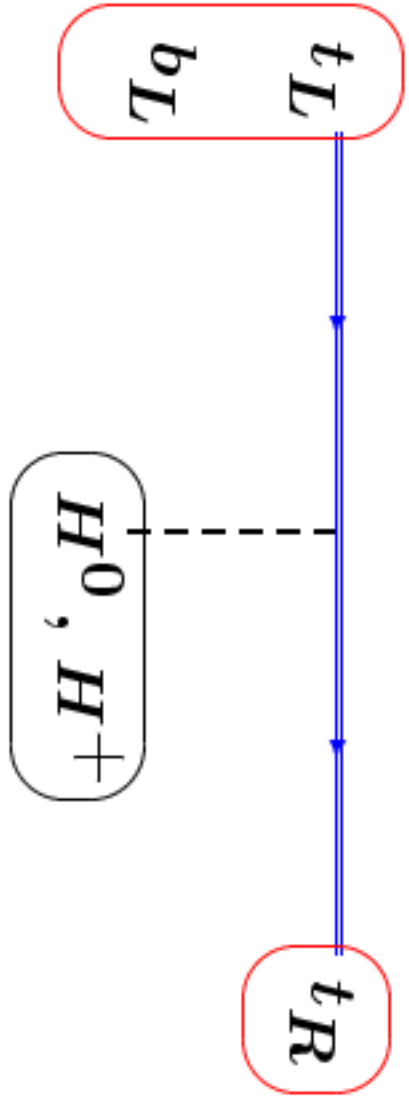
spontaneously broken by the VEVs of scalars transforming as  $(3, \bar{3}, 1, \dots, 1), \dots$

The coupling of the top quark to the Higgs field changes with the distance (similar to vacuum polarization in electrodynamics).

$$\lambda_t \bar{t}_R \langle H^0 \rangle t_L, \quad \langle H^0 \rangle \approx 174 \text{ GeV}$$

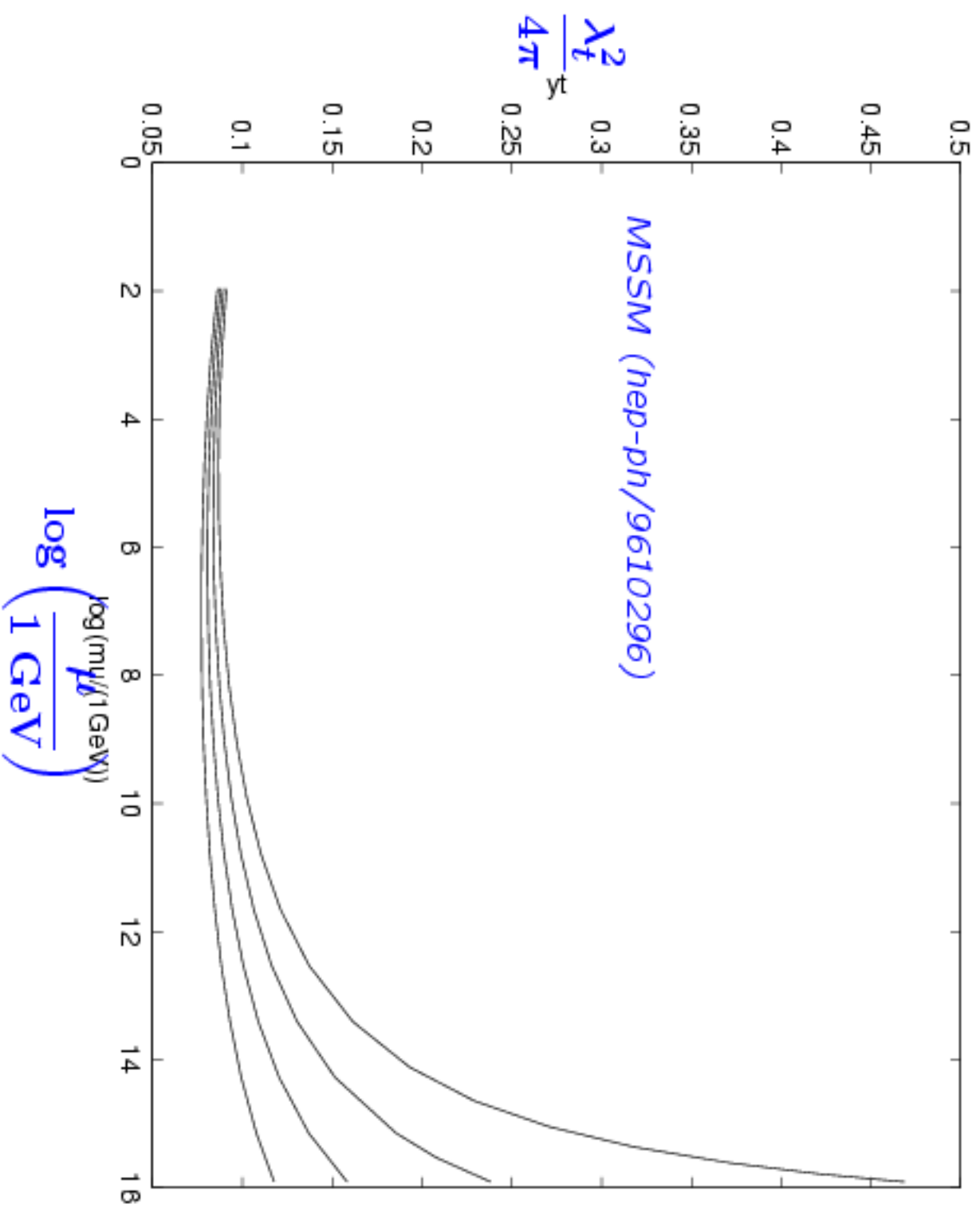
In a world of only top and Higgs:

$$\lambda_t(\mu) = \frac{\lambda_t(m_t)}{\sqrt{1 - \frac{9\lambda_t^2(m_t)}{64\pi^2} \ln \frac{\mu}{m_t}}}$$



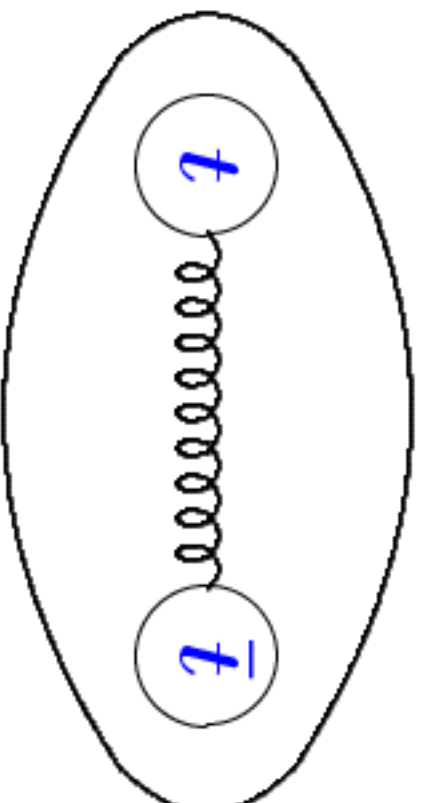
# Infrared Fixed Point for $\lambda_t$

(C.T. Hill, 1981, ...)



**Top condensation  $\Rightarrow$  Higgs boson is a  $t\bar{t}$  bound state!**  
(Bardeen, Hill, Lindner, 1990, ...)

*Binding may be due to some strongly-interacting heavy gauge bosons*



**New heavy quarks (vectorlike) could accelerate the  $\lambda_t$  running:  
scale of Higgs compositeness may be as low as a few TeV.**

*Explicit models: top seesaw, QCD in extra dimensions, ...*

# Is there a Higgs boson?

## EW/SB by boundary conditions

*Csaki, Grojean, Pilo, Terning: hep-ph/0308038*

*Cacciapaglia, Csaki, Marandella, Terning, hep-ph/0607146*

One warped ED, bulk  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge group broken by boundary conditions.

AdS dual of a (walking) technicolor-like theory, (the presence of the IR brane breaks electroweak symmetry)

- lightest  $W$ ,  $Z$  and photon resonances around 1.2 TeV
- no fundamental (or composite) Higgs boson

## What to look for

- Vector-like fermions (KK modes)
- New gauge bosons  
(*e.g.*, unitarity restored by  $Z'$ ,  $W'$ , ...)

- extended Higgs sectors  
(*e.g.*, radion-Higgs mixing, two composite  $H$ , ... )

⇒ **many possibilities!** *Will the experiments be able to differentiate between models?*

## Conclusions

- There are various types of extra spatial dimensions, depending on the metric (flat vs. warped), which fields propagate in the bulk, type of compactification, etc.
- Phenomenological implications are highly model dependent. Even closely related theories, such as one versus two universal extra dimensions, predict quite different signals.
- A warped extra dimension provides a nice explanation for the hierarchy problem.
- Any particle that propagates in  $D \geq 5$  would appear in experiments as a tower of heavy 4-dimensional particles. There exists purely 4D theories with similar spectra and interactions, which are interesting whether or not extra dimensions exist in nature!